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1.

$$\lim_{n\to\infty} \frac{(n+1)(n+2)(n+3)}{n^4+1} = \lim_{n\to\infty} \frac{n^3+6n^2+11n+6}{n^4+1} \overset{n^4}{=} \frac{0}{1} = 0$$

2.

$$\lim_{n\to\infty} \frac{n+2n^2+n^3}{(n+1)^3+(n+2)^2+(n+3)} = \lim_{n\to\infty} \frac{n+2n^2+n^3}{n^3+4n^2+8n+8} \overset{n^3}{=} \frac{1}{1} = 1$$

3.

$$\lim_{n\to\infty} \sqrt{\frac{n+\sqrt{n^2+\sqrt{n^4+1}}}{n+1}} \overset{\sqrt{n}}{=} \frac{\sqrt{1+\sqrt{1+\sqrt{1}}}}{\sqrt{1}} = \sqrt{1+\sqrt{2}}$$

4.

$$\text{UP: } \lim_{n\to\infty} a_n = 0, \lim_{n\to\infty} b_n = 0, \lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

$$a_n = \frac{1}{n}, b_n = \frac{1}{n^2} \Rightarrow \frac{a_n}{b_n} = \frac{\frac{1}{n}}{\frac{1}{n^2}} = n$$

5.

$$(-1)^n \frac{n}{n+1}$$

$$n = 2k,\ k \in \mathbb{N} : \lim_{k\to\infty} (-1)^{2k} \frac{2k}{2k+1} = \lim_{k\to\infty} \frac{2k}{2k+1} = 1$$

$$n = 2k+1,\ k \in \mathbb{N}_0 : \lim_{k\to\infty} (-1)^{2k+1} \frac{2k+1}{2k+2} = \lim_{k\to\infty} -\frac{2k+1}{2k+2} = -1$$

$$\limsup_{n\to\infty} (-1)^n \frac{n}{n+1} = 1$$

$$\liminf_{n\to\infty} (-1)^n \frac{n}{n+1} = -1$$

6.

$$\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}$$

$$n = 2k,\ k \in \mathbb{N} : \lim_{k\to\infty} \left[\frac{(-1)^{2k}}{2k} + \frac{1+(-1)^{2k}}{2}\right] = \left\|\frac{1}{\infty} + \frac{1+1}{2}\right\| = 1$$

$$n = 2k+1,\ k \in \mathbb{N}_0 : \lim_{k\to\infty} \left[\frac{(-1)^{2k+1}}{2k+1} + \frac{1+(-1)^{2k+1}}{2}\right] = \left\|\frac{-1}{\infty} + \frac{1-1}{2}\right\| = 0$$

$$\limsup_{n\to\infty} \left[\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}\right] = 1$$

$$\liminf_{n\to\infty} \left[\frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}\right] = 0$$