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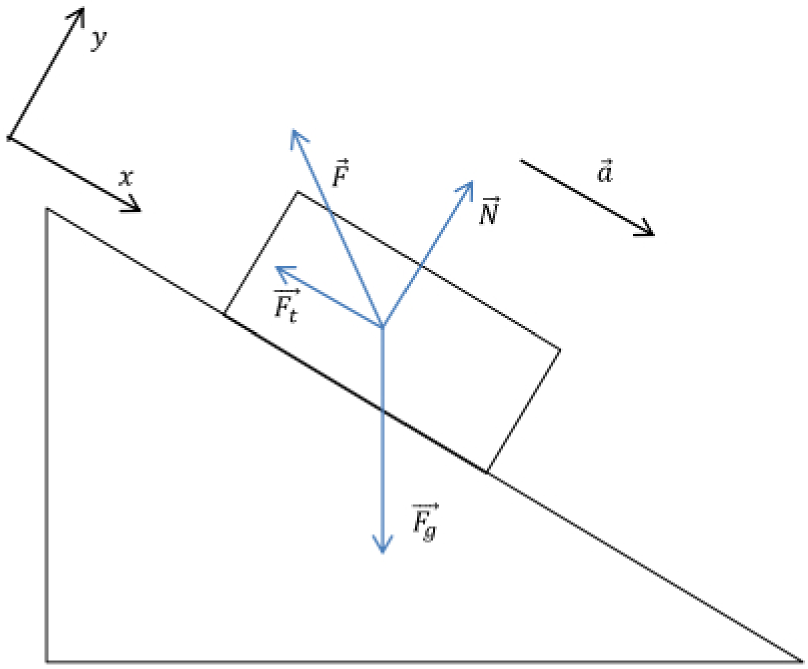
8. cvičení

1.
Dáno:
$m = 100$ kg
$\alpha = 30°$ kg
$a = 1$ ms^{-2}
$f = 0,01$
$l = 5$ m
$h = 3$ m
$\beta = \arcsin \frac{l}{h} = \arcsin 0,6$

Rozepíšem vektory do zvoleného souřadnicového systému a pomocí nich zapíšeme 2.NZ:



$\overrightarrow{a} = (a, 0)$
$\overrightarrow{F_g} = (mg \sin \beta, -mg \cos \beta)$
$\overrightarrow{N} = (0, N)$
$\overrightarrow{F_t} = (-Nf, 0)$
$\overrightarrow{F} = (-F \cos \alpha, F \sin \alpha)$

$$
\begin{aligned}
\overrightarrow{F_g} + \overrightarrow{N} + \overrightarrow{F_t} + \overrightarrow{F} &= m\vec{a} \\
y: \quad -mg\cos\beta + N + F\sin\alpha &= 0 \\
N &= mg\cos\beta - F\sin\alpha \\
x: \quad mg\sin\beta - Nf - F\cos\alpha &= ma \\
mg\sin\beta - fmg\cos\beta + fF\sin\alpha - F\cos\alpha &= ma \\
F(f\sin\alpha - \cos\alpha) &= ma + mg(f\cos\beta - \sin\beta) \\
F &= \frac{ma + mg(f\cos\beta - \sin\beta)}{f\sin\alpha - \cos\alpha} \\
F &= \frac{100 + 100\cdot 9,81(0,01\cdot\cos\arcsin 0,6 - 0,6)}{0,01\sin 30^\circ - \cos 30^\circ}\ \mathrm{N} = 558\ \mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\vec{v} &= \int \vec{a}\,\mathrm{d}t = (at + c_1, c_2) \\
\vec{v}(0) &= \vec{0} \Rightarrow c_1 = 0 \wedge c_2 = 0 \Rightarrow \vec{v} = (at, 0) \\
\vec{r} &= \int \vec{v}\,\mathrm{d}t = \left(\frac{1}{2}at^2 + d_1, d_2\right) \\
\vec{r}(0) &= \vec{0} \Rightarrow d_1 = 0 \wedge d_2 = 0 \Rightarrow \vec{v} = \left(\frac{1}{2}at^2, 0\right) \\
l = r(t_1) &= \frac{1}{2}at_1^2 \Rightarrow t_1 = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2\cdot 5}{1}}\ \mathrm{s} = \sqrt{10}\ \mathrm{s}
\end{aligned}
$$

a)

$$
\begin{aligned}
W_{\overrightarrow{F}} &= \int_{t_0}^{t_1} \overrightarrow{F}\cdot\vec{v}\,\mathrm{d}t = \int_{t_0}^{t_1} -Fat\cos\alpha\,\mathrm{d}t = -Fa\cos\alpha\left[\frac{t^2}{2}\right]_{t_0=0}^{t_1=\sqrt{\frac{2l}{a}}} = -Fa\cos\alpha\frac{2l}{2a} \\
&= -Fl\cos\alpha = -558\cdot 5\cdot\cos 30^\circ\ \mathrm{J} = -2416\ \mathrm{J}
\end{aligned}
$$

b)

$$
\begin{aligned}
W_{\overrightarrow{F_g}} &= \int_{t_0}^{t_1} \overrightarrow{F_g}\cdot\vec{v}\,\mathrm{d}t = \int_{t_0}^{t_1} mgat\sin\beta\,\mathrm{d} = mga\sin\beta\left[\frac{t^2}{2}\right]_{t_0=0}^{t_1=\sqrt{\frac{2l}{a}}} = mga\sin\beta\frac{2l}{2a} \\
&= mgl\sin\beta = 100\cdot 9,81\cdot 5\cdot 0,6\ \mathrm{J} = 2943\ \mathrm{J}
\end{aligned}
$$

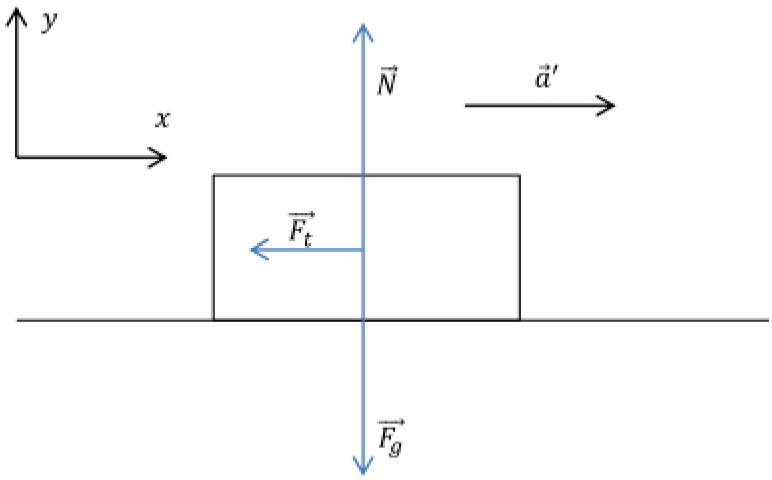
c)

$$
\begin{aligned}
W_{\overrightarrow{F_t}} &= \int_{t_0}^{t_1} \overrightarrow{F_t}\cdot\vec{v}\,\mathrm{d}t = \int_{t_0}^{t_1} -fNat\,\mathrm{d}t = \int_{t_0}^{t_1} -f(mg\cos\beta - F\sin\alpha)at\,\mathrm{d}t \\
&= -f(mg\cos\beta - F\sin\alpha)a\left[\frac{t^2}{2}\right]_{t_0=0}^{t_1=\sqrt{\frac{2l}{a}}} = -f(mg\cos\beta - F\sin\alpha)a\frac{2l}{2a} \\
&= -fl(mg\cos\beta - F\sin\alpha) = -0,01\cdot 5\cdot(100\cdot 9,81\cos\arcsin 0,6 - 558\sin 30^\circ) = -25,3\ \mathrm{J}
\end{aligned}
$$

d)

$$
\begin{aligned}
\overrightarrow{F_v} &= m\vec{a} \\
W_{\overrightarrow{F_v}} &= \int_{t_0}^{t_1} \overrightarrow{F_v} \cdot \vec{v}\,\mathrm{d}t = \int_{t_0}^{t_1} m\vec{a} \cdot \vec{v}\,\mathrm{d}t = \int_{t_0}^{t_1} ma^2 t\,\mathrm{d}t = ma^2 \left[\frac{t^2}{2}\right]_{t_0=0}^{t_1=\sqrt{\frac{2l}{a}}} \\
&= ma^2 \frac{2l}{2a} = mal = 100 \cdot 1 \cdot 5\ \mathrm{J} = 500\ \mathrm{J}
\end{aligned}
$$

Při vodorovném pohybu znovu napíšeme 2.NZ:



$$
\begin{aligned}
\overrightarrow{F_g} + \overrightarrow{N} + \overrightarrow{F_t} &= m\vec{a}' \\
y: \quad N &= mg \\
x: \quad ma' &= -Nf = -fmg \Rightarrow a' = -fg \\
v_1 &= at_1 = \sqrt{10}\ \mathrm{ms}^{-1} \\
v(t') &= v_1 + a't' = 0 \Rightarrow t' = \frac{v_1}{-a'} \\
s &= v_1 t' + \frac{1}{2}a't'^2 = \frac{v_1^2}{-a'} + \frac{1}{2}\frac{v_1^2}{a'} = -\frac{1}{2}\frac{v_1^2}{a'} = \frac{1}{2}\frac{v_1^2}{fg} = \frac{1}{2}\frac{10}{0,01 \cdot 9,81}\ \mathrm{m} = 51\ \mathrm{m}
\end{aligned}
$$

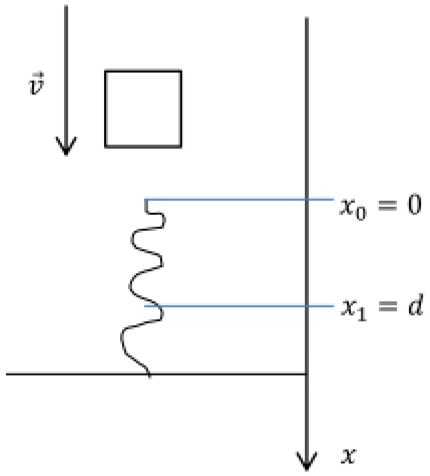
2. HRW, kapitola 7, 40Ú

Dáno:

$m = 250$ g $= 0,25$ kg

$k = 2,5$ Ncm^{-1} $= 2,5 \cdot 10^2$ Nm^{-1}

$A = d = 12$ cm $= 12 \cdot 10^{-2}$ m



a)

$$W_{\overrightarrow{F_G}} = \Delta E_p = mg(x_1 - x_0) = mgd = 0,25 \cdot 9,81 \cdot 12 \cdot 10^{-2}\ \text{J} = 0,3\ \text{J}$$

b)

$$W_{\overrightarrow{F_p}} = \frac{1}{2}k \cdot (x_0^2 - x_1^2) = -\frac{1}{2}kd^2 = -\frac{1}{2} \cdot 2,5 \cdot 10^2 \cdot (12 \cdot 10^{-2})^2\ \text{J} = -1,8\ \text{J}$$

c)

Během kmitů platí zákon zachování mechanické energie:

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{1}{2}kd^2 - mgd \\ v^2 &= \frac{kd^2}{m} - 2gd \\ v &= \sqrt{\frac{kd^2}{m} - 2gd} \\ v &= \sqrt{\frac{2,5 \cdot 10^2 \cdot (12 \cdot 10^{-2})^2}{0,25} - 2 \cdot 9,81 \cdot 12 \cdot 10^{-2}}\ \text{ms}^{-1} = 3,47\ \text{ms}^{-1} \end{aligned}$$

d)
$v' = 4v$

$$\begin{aligned}
\frac{1}{2}m(4v)^2 &= \frac{1}{2}kd'^2 - mgd' \\
\frac{1}{2}kd'^2 - mgd' - 2mv^2 &= 0 \\
D &= m^2g^2 + 4kmv^2 \\
d' &= \frac{mg \pm \sqrt{m^2g^2 + 4kmv^2}}{k} \\
\text{protože } \sqrt{m^2g^2 + 4kmv^2} &> mg \text{ musíme vzít jen výraz s } + \\
d' &= \frac{mg + \sqrt{m^2g^2 + 4kmv^2}}{k} \\
d' &= \frac{0,25 \cdot 9,81 + \sqrt{0,25^2 \cdot 9,81^2 + 4 \cdot 2,5 \cdot 10^2 \cdot 0,25 \cdot 3,47^2}}{2,5 \cdot 10^2}\ \text{m} \\
d' &= 23 \cdot 10^{-2}\ \text{m} \\
\frac{d'}{d} &= \frac{23 \cdot 10^{-2}\ \text{m}}{12 \cdot 10^{-2}\ \text{m}} = 1,9
\end{aligned}$$